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Poisson probability distribution analysis of Makurdi and Abeokuta rainfalls

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Abstract

Early information for sustainable utilization of water resources through poisson probability distribution of rainfall is an important regulatory measure for flood control and water security management. As a follow-up to our previous studies on distributions, this paper reports statistical goodness-of-fit evaluations of selected rainfall data. It is the utilization of the maximum likelihood method (MLM) for the poisson probability distribution (PPD) of selected rainfall data. The numerically estimated constant of the density of PPD was estimated by the MLM, and Microsoft Excel Solver (MES). These estimated constants were used to compute probabilities of poisson distributions. The computed probabilities using the constants obtained were evaluated statistically (analysis of variance, (ANOVA), relative error, model of' selection criterion (MSC), Coefficient of Determination (CD) and Correlation coefficient (R). The study established that the poisson probability distribution's parameter (p) was the average of the logarithm to base 10 of rainfall using the MLM and MES estimators. The constants were found to be 0.665 and 0.535 for Makurdi, 0.695 and 0.478 for Abeokuta using MLM and MES, respectively. The relative errors were found to be 0.479 and 0.743, and 1.141 and 1.509 for Makurdi and Abeokuta using MLM and MES, respectively. The correlation coefficient for Makurdi and Abeokuta using MLM and MES were found to be 0.876 and 0.800, and 0.269 and 0.341, respectively. It was concluded that the MLM constant was better than MES based on the values of MSC, CD, relative error and R. MLM predicted Weibull probability of rainfall intensity better than MES. Utilization of PPD in the estimation of rainfall intensity will help in the prediction of rainfall for agriculture in attaining Sustainable Development Goal 2 (zero hunger), regulatory measures for flood control and water security management. There is a need to evaluate MLM and other probability distributions.

Keywords: Poisson distribution, rainfall intensity, the goodness of fit test, maximum likelihood method, Microsoft Excel Solver

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1 Introduction

The risk of floods has become a grave concern worldwide. Floods have affected a supreme number of people and have the extreme damage possible of all-natural disasters worldwide (Al-Zahrani, 2018). With more life-threatening weather patterns forecast in the future, and with an increase in population progress and urban areas development, more recurrent floods are predicted to occur. These floods and climate changes will meaningfully affect the hydrological comportment and response of many areas to storm events of the higher magnitude of runoff and higher return of small floods (Al-Zahrani, 2018). Fig. 1 confirms the effect of floods on the environment as surface water pollution carriers. Fig. 2 presents the influence of climate change on the theoretical design compromise for typical semi-urban and urban infrastructures.

The speedy urban growth in many geopolitical zones of Nigeria has converted the surface appearances of undisturbed soils into semi-permeable or impermeable layers of asphalts and concretes. Present land use renovations have significantly enhanced the potential of these cities to create relatively large amounts of surface runoff from rainfall intensity events, causing extraordinary urban and semi-urban floodings which endanger achievement of sustainable goal 2 (zero hunger), goal 15 (life on land) and goal 14 (life below water). With reference to the expected risks associated with rainfall, and floods, much research has been conducted to simulate and analyse rainfall intensity duration in different parts and regions of the world.

However, cities (such as Abeokuta and Makurdi) in Nigeria had undergone speedy urban development that meaningfully affected their land covers. These cities are also located in geopolitical zones with extreme climate conditions. These rapid urbanizations and the extreme rainfall intensities characteristics of these cities are expected to meaningfully affect the runoff progression and the amount of storm and rainfall runoff during these floods. It is clearly expected that these changes in urbanization and land covers will be altered and alter the hydrologic response of the watershed. Therefore, there is a need for further studies on flooding potential through probability distributions of rainfall intensities duration and maximum likelihood estimation under the circumstances, a numeral of empirical relationships have been utilized for the probability of rainfall intensity in hydrology, water resources areas, civil and environmental engineering. Numerous probability distributions are extensively in use over the past three decades for modelling rainfall intensity data in areas of research such as environment, reliability, economics, engineering, biological studies, demography and medical sciences (Dikko and Faisal, 2018). Two characteristic probability distribution expressions and functions used for rainfall intensity duration data analysis are the exponential and the Weibull probability density functions (Pieracci, 1997). These probability distributions are alienated into two portions as follows (Fiondella and Zeephongsekul, 2015; Gao et al., 2020; Jones et al., 2020):

(a) Discrete Probability Distributions (Binomial Distribution, Bernoulli Distribution and Poisson Distribution)

(b) Continuous Probability Distributions (Normal Distribution, Continuous Uniform Distribution, Log-Normal Distribution and Exponential Distribution)

There are ten estimation methods to estimate the reliability of the distribution. These methods are: MLM, Least square and weighted least square estimation, Percentile estimation, Maximum product of estimation, Minimum spacing distance estimation, Crame´r-Von Mises estimation, Anderson-Darling and Right-tail Anderson-Darling estimation (Almarashi et al., 2020). Literature provides information on probability distributions (Weibull, Normal, and log-normal), but there is little or no information on Maximum likelihood estimation and Poisson probability distribution (PPD). With this advancement in computer applications and technologies, which makes it possible to collect rainfall intensity-duration data at various stations there is a need to utilize the maximum likelihood method (MLM) and PPD for rainfall intensity-duration data analysis. This study, therefore focuses on the utilization and evaluation of maximum likelihood estimation and PPD for rainfall intensity-duration data analysis, which will help in attaining sustainable development goals 2 (zero hunger), goal 15 (life on land) and goal 14 (life below water), help in regulatory measure for flood control and water security management.

2 Materials and Methods

2.1 Data collection and analysis

Rainfall intensity-duration data of two stations (Abeokuta (1986 to 2010) and Makurdi (1979 to 2009)) were collected from David et al. (2019) and Isikwue et al. (2012). The data were analysed statistically using analysis of variance (ANOVA). The probability of the rainfall intensity was computed using Weibull probability mathematical expression as follows (Teyabeen et al., 2017; Almarashi et al., 2020) and Equations 1 and Equation 2):

$$T_m(x) = \frac{n+1}{m} \tag{1}$$

where $_{T}m$ is the return period, n is the sample size and m is the rank.

$$f(x) = p_m(x) = \frac{1}{T_m} \tag{2}$$

where $p_m(x)$ is the theoretical probability (probability index) and $f(x)$ is the cumulative probability.

The Weibull distribution is the most preferred in modelling the rainfall intensity data. The parameter of the PPD was calculated using the MLM and Microsoft Excel Solver (MES). The calculated PPD's parameter (MLM and MES methods) was used to establish the PPD were evaluated statistically using analysis of variance ANOVA), Relative error, Model

Figure 1. (a) Run off with recalcitrant materials as floating solids on Opa river in Ile -Ife, Nigeria, (b) Combination of Run off with recalcitrant materials as floating solids on Opa river in Ile -Ife, Nigeria, (c) Effects of floods in a Flooded Communities of Markurdi, Nigeria, and (d) Significant effects of floods in a community in Ile -Ife, Nigeria

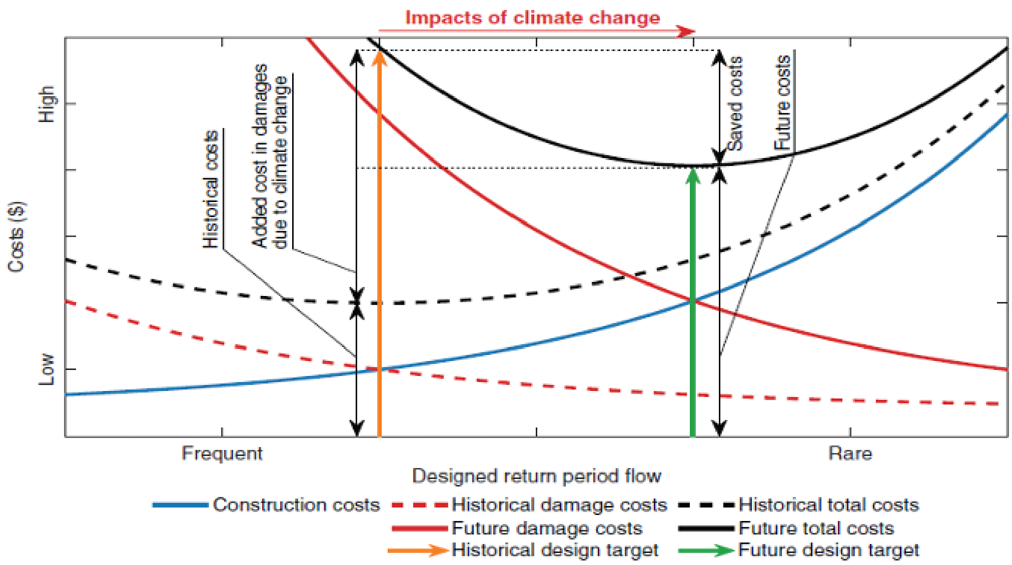

Figure 2. Impact of climate change on the theoretical design compromise for typical urban infrastructure (Source: Martel et al. (2021))

of selection criterion (MSC), Coefficient of Determination (CD) and Correlation coefficient (R). MSC indicates higher exactness, cogency and a good fit of the methods. MSC was calculated using Equation 3 as follows (Adekunbi et al., 2020):

$$MSC = ln \left[\frac{\sum_{i=1}^{n} (Y_{obsi} - \overline{Y}_{obs})^2}{\sum_{i=1}^{n} (Y_{obsi} - Y_{cali})^2} \right] - \frac{2p}{n} \quad (3)$$

where Y_{obsi} is the probability value using Weibull probability mathematical expression; is the average probability value using Weibull probability mathematical expression; p is the total number of fixed parameters to be estimated in the methods; n is the total number of rainfall intensities calculated, and Y_{cali} is the probability calculated using the MLM estimator.

The coefficient of determination (CD) can be understood as the quantity of expected data variation that can be described by the obtained data. Higher values of CD indicate higher accurateness, cogency and good fitness of the device. CD, correlation coefficient and relative error can be expressed as follows (Equations 4, 5, and 6):

$$CD = \frac{\sum_{i=1}^{n} (Y_{obsi} - \overline{Y}_{cali})^2 - \sum_{i=1}^{n} (Y_{obsi} - Y_{cali})^2}{\sum_{i=1}^{n} (Y_{obsi} - \overline{Y}_{cali})^2} \quad (4)$$

where $\overline{Y}_{cali}$ is the average probability value calculated using the MLM estimator.

$$R = \sqrt{\frac{\sum_{i=1}^{n} (Y_{obsi} - \overline{Y}_{cali})^2 - \sum_{i=1}^{n} (Y_{obsi} - Y_{cali})^2}{\sum_{i=1}^{n} (Y_{obsi} - \overline{Y}_{cali})^2}} \quad (5)$$

$$R_{ei}(\%) = \left(\frac{1}{N}\right) \sum_{i=1}^{N} \left(\frac{Y_{obsi} - Y_{cali}}{Y_{obsi}}\right) \quad (6)$$

Fig. 3 presents the summary of the Microsoft Excel Solver procedures. MES was used for the determination of these empirically derived parameters based on availability at no additional cost. The procedure used for the Microsoft Excel solver can be summarized as follows: (i) Excel solver was added in Microsoft Excel, (ii) Target of the numerical analysis

$$\left((K_p - K_t)^2 = 0 \right)$$

operation and changing cells were set, where K_p is the probability value using Weibull mathematical expression

$$(f(x)) = p_m(x) = \frac{1}{T_m}$$

and K_t is the PPD probability calculated using MLM

$$f(f) = \frac{\lambda^x}{x!} exp^{-\lambda} \, ;$$

and (iii) Microsoft Excel Solver was allowed to iterate at 200 iterations with 0.005 tolerance.

3 Results and Discussion

3.1 Rainfall intensities data

Fig. 4 and Fig. 5 present the rainfall intensity data from David et al. (2019), while Fig. 6 confirms the rainfall intensity data from Isikwue et al. (2012). From these figures, the highest rainfall-duration-intensity frequency occurred when the duration was 5 min in the year 1 (1979, Isikwue et al. (2012) and 1986, David et al. (2019), respectively) and the lowest rainfall-duration intensity frequency occurred when the duration was 1440 min in the 30th year (2009, Isikwue et al. (2012) and 2010, David et al. (2019)). Table 1 confirms the result of an ANOVA of the rainfall-duration intensity frequency (Abeokuta) with respect to the years.

From Table 1, the $F_{24,300}$ = 1.652 and p = 3.02 × 10^{-2} for analysis of the rainfall-duration intensity frequency between the years. This result established that there were significant differences between rainfall-duration-intensity frequency values within the years at a 95% confidence level ($p < 0.05$). Table 2presents the outputs from an ANOVA of rainfall-duration-intensity frequency within the duration of the rainfall. The Table confirms that the $F_{12,312}$ = 84.32 and p = 2.47 × 10^{-90} for analysis of the rainfall-duration-intensity frequency between the duration of the rainfall. This result established that there was a significant difference between rainfall-duration-intensity frequency values within these durations at a 95% confidence level ($p < 0.05$). Table 3 confirms the result of an ANOVA of the rainfall-duration intensity frequency (Makurdi) with respect to the return period. From Table 3, the $FF_{5,204}$ = 120.59 and p = 6.21 × 10^{-59} for analysis of the rainfall-duration intensity frequency between the return years. This result established that there were significant differences between rainfall-duration-intensity frequency values within the years at a 95% confidence level ($p < 0.05$). Table 4 confirms the outputs from an ANOVA of rainfall-duration-intensity frequency within the duration of the rainfall. Table 4 confirms that the $FF_{34,175}$ = 0.926 and p = 5.90 × 10^{-1} for analysis of the rainfall-duration-intensity frequency between the duration of the rainfall. This result established that there was no significant difference between rainfall-duration-intensity frequency values within these durations at a 95% confidence level ($p > 0.05$).

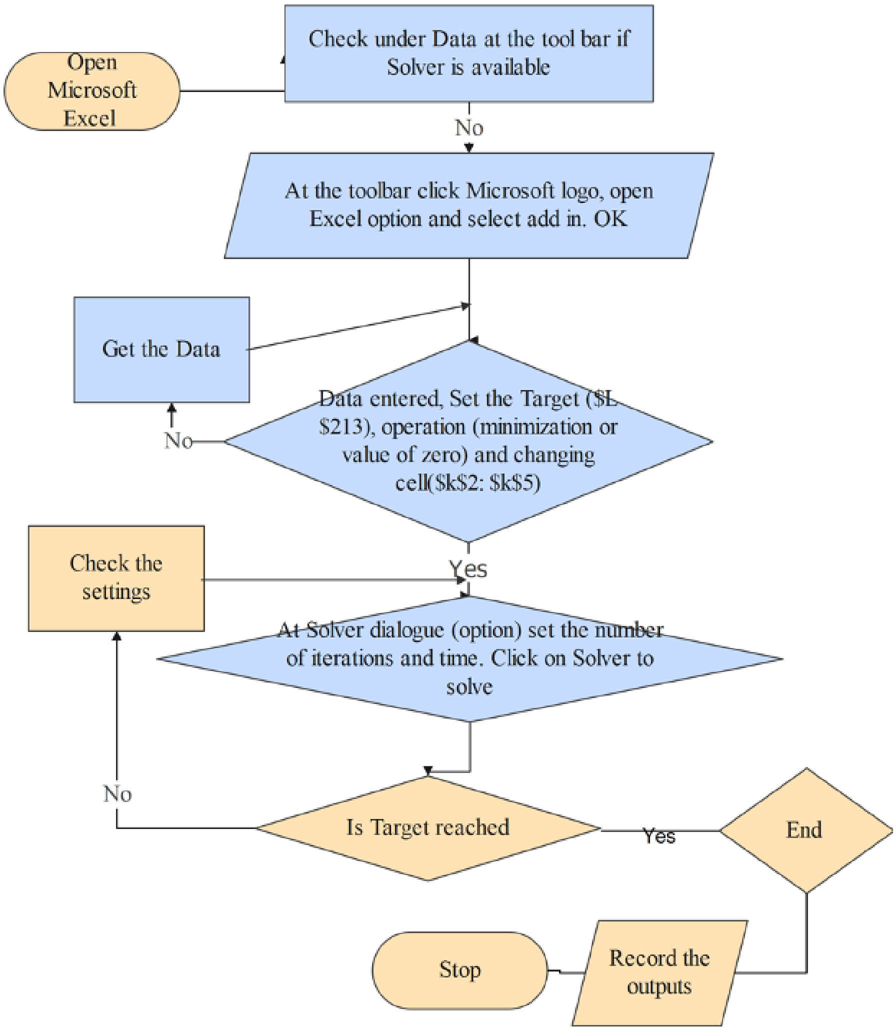

Figure 3. Summary of the Microsoft Excel Solver Procedures

Table 5 presents the statistical properties (average, maximum, minimum, standard deviation and Skewness) of the rainfall intensity data in respect of Abeokuta. Table 6 confirms the statistical summary (average, maximum, minimum, and standard deviation) of the rainfall intensity data for Makurdi. From Table 5, the averages of rainfall intensity for Abeokuta were found to be 206.40, 164.54, 135.14, 117.83, 85.64, 69.06, 55.33, 41.23, 31.72, 22.81, 19.02, 15.78 and 11.59 mm/h for duration times of 10, 20, 30, 45,5, 60, 90, 120, 15, 180, 240, 300 and 420 minutes, respectively. These results confirmed that heavy rainfalls had the lowest duration and the lowest rainfall intensities had the highest duration. The other statistical properties (maximum, minimum and standard deviation) followed the same trend as the averages.

From Table 5, the Skewness of the rainfall intensities was between 0.16 and 1.32. All these durations had positive Skewness, which indicated that most of the values of these rainfall intensities concentrated on the right of the mean, with extreme values to the left. Table 6 presents the average, maximum, minimum and standard deviation of the rainfall intensities for Makurdi at different return periods. The averages of the rainfall intensities from Makurdi were found to be 180.541, 69.320, 36.819, 22.815, and 12.119 mm/h for the return period of 100, 50, 25, 10, 5 and 2 years, respectively. These trends of rainfall intensities for Tables 5 and 6 agreed with literature (Abouammoh, 1991; Gupta and Huang, 2014; Hassan et al., 2017; Lihou and Spence, 1988; Madsen et al., 2017; Mahmoudi and Sepahdar, 2013; Tramblay et al., 2013; Wagh and Kamalja, 2015; Francesco et al., 2014; Morales and Vicini, 2020; Aleksandrovskaya et al., 2019; Jeong et al., 2017).

3.2 Derivation of Poisson parameter

The log-likelihood function of this random sample is given as follows (Couton and Danech-Pajouh, 1997; Brosius, 2015; Sakai et al., 2018):

$$L(x_1, x_2, x_3, \ldots, x_n) = \sum_{i=1}^{n} lnf(x_1, \theta) \qquad (7)$$

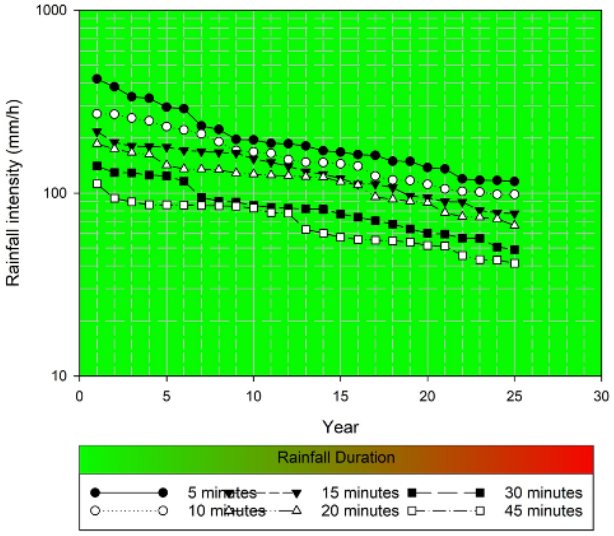

Figure 4. Rainfall intensity of Abeokuta (duration of between 5 and 45 min)

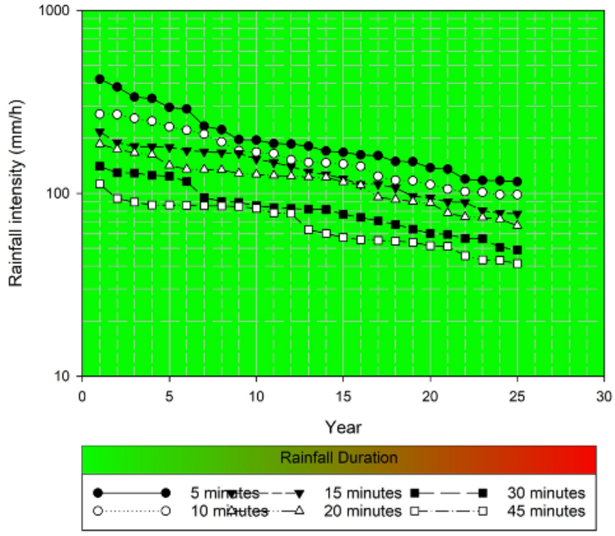

Figure 5. Rainfall intensity of Abeokuta (duration of between 60 and 420 min)

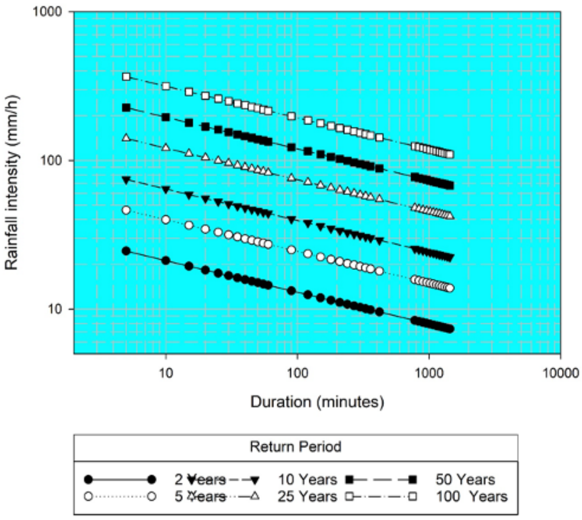

Figure 6. Rainfall intensity of Makurdi (return period of between 2 and 100 years)

Table 1. Result of an ANOVA of the rainfall-duration intensity frequency (Abeokuta) with respect to the years

SoV	SS	Df	MSS	F-Value	P-value
Between years	181955	24	7581.458	1.651739	0.030253
Within years	1376996	300	4589.985		
Total	1558951	324			

SoV: Sources of variation; SS: Df: degrees of freedom; Sum of square; MSS: Mean sum of square

Table 2. Outputs from an ANOVA of rainfall-duration-intensity frequency within the duration of the rainfall

SoV	SS	Df	MSS	F-Value	P-value
Between RID	1191745	12	99312.08	84.38	2.47×10^{-90}
Within RID	367205.6	312	1176.941		
Total	1558951	324			

SoV: Sources of variation; SS: Df: degrees of freedom; Sum of square; MSS: Mean sum of square

Table 3. Result of an ANOVA of the rainfall-duration intensity frequency (Makurdi) with respect to the return period

SoV	SS	Df	MSS	F-Value	P-value
Between return periods	721710.3	5	144342.1	120.5911	6.21×10^{-59}
Within return periods	244178.7	204	1196.954		
Total	965889	209			

SoV: Sources of variation; SS: Df: degrees of freedom; Sum of square; MSS: Mean sum of square

Table 4. Outputs from an ANOVA of rainfall-duration-intensity frequency within the duration of the rainfall

SoV	SS	Df	MSS	F-Value	P-value
Between durations	147235.3	34	4330.449	0.925701	0.590208
Within durations	818653.7	175	4678.021		
Total	965889	209			

SoV: Sources of variation; SS: Df: degrees of freedom; Sum of square; MSS: Mean sum of square

Table 5. Statistical properties (average, maximum, minimum, standard deviation and Skewness) of the rainfall intensity data in respect of Abeokuta

Duration	Average	Median	Geomean	Standard deviation	Skewness
5	206.404	175.8	166.509	86.802	1.141
10	164.54	147.25	139.916	57.502	0.628
15	135.144	129.2	118.85	40.658	0.159
20	117.832	122.7	105.85	33.575	0.249
30	85.644	81.8	78.617	27.277	0.64
45	69.012	61.75	65.328	19.665	0.264
60	55.328	58.45	53.413	16.083	0.505
90	41.232	40.8	41.165	10.941	0.527
120	31.72	30.65	32.338	8.682	0.949
180	22.812	21.35	23.912	6.462	1.322
240	19.024	16.7	20.324	5.282	1.196
300	15.784	14.75	17.184	4.121	1.075
420	11.592	11.2	12.99	2.772	0.999

Table 6. Statistical summary (average, maximum, minimum, and standard deviation) of the rainfall intensity data for Makurdi

Return period	Average	Median	Geomean	Standard deviation	Skewness
2	12.119	10.635	10.854	4.506	0.928
5	22.815	20.02	20.595	8.484	0.928
10	36.819	32.315	33.435	13.692	0.928
25	69.32	60.83	63.446	25.777	0.928
50	111.87	98.17	103.003	41.599	0.928
100	180.541	158.435	167.224	67.135	0.928

The MLM is produced as follows (Pichugina, 2008; Pobočíková et al., 2017):

$$L(\theta) = \prod_{i=1}^{n} fx(x_1, \theta) \tag{8}$$

Take the natural log of the likelihood, collect terms involving θ.

$$ln(L(\theta)) = ln\left[\prod_{i=1}^{n} fx(x_1, \theta)\right] \tag{9}$$

Find the value of $\theta\epsilon\theta$, θ, for which log $L(\theta)$ is maximized by differentiation.

$$\frac{d}{d\theta}\left[ln(L(\theta))\right] = \frac{d}{d\theta}\left\{ln\left[\prod_{i=1}^{n} fx(x_1, \theta)\right]\right\} \tag{10}$$

In the parameter (θ). If θ is vector-valued, say $\theta = (\theta_i, \ldots, \theta_n)$, then find $\theta = (\theta_i, \ldots, \theta_n)$ by simultaneously solving the n equations given by other researchers (Yashunsky, 2019; Wentzel-Larsen and Anhøj, 2019; Pham and Pham, 2019; Piast, 2019; Pichon, 2018; Roberts, 2019; Santos, 2018; Sulbhewar and Raveendranath, 2017; Telles, 2020; Hosseinzadehtalaei et al., 2020).

$$\frac{\partial}{\partial\theta_j}\left[ln(L(\theta))\right] = \frac{\partial}{\partial\theta}\left\{ln\left[\prod_{i=1}^{n} fx(x_1, \theta)\right]\right\} = 0; \quad j = 1\ldots k \tag{11}$$

Poisson probability distribution can be stated as follows (Yashunsky, 2019; Wentzel-Larsen and Anhøj, 2019; Pham and Pham, 2019; Piast, 2019; Pichon, 2018; Roberts, 2019; Santos, 2018; Sulbhewar and Raveendranath, 2017; Telles, 2020; Hosseinzadehtalaei et al., 2020):

$$f(x) = \frac{\lambda^x}{x!} exp^{-\lambda} \tag{12}$$

$$L(x) = \prod_{i=1}^{n} fx(x_i, \lambda) = \frac{\lambda^{nx}}{x^n!} exp^{-n\lambda} \tag{13}$$

$$\begin{aligned} ln\left[L(x)\right] = ln\left[\prod_{i=1}^{n} fx(x_i, \lambda)\right] = \\ nxln(\lambda) - n\lambda - \sum_{i=1}^{n} ln(x_i!) \end{aligned} \tag{14}$$

$$\begin{aligned} \frac{\partial}{\partial\lambda}\left[nxln(\lambda) - n\lambda - \sum_{i=1}^{n} ln(x_i!)\right] = \\ \frac{\partial}{\partial\lambda}\left[\sum_{i=1}^{n} ln(\lambda) - n\lambda - \sum_{i=1}^{n} ln(x_i!)\right] \end{aligned} \tag{15}$$

$$\begin{aligned} \frac{\partial}{\partial\lambda}\left[\sum_{i=1}^{n} x_i ln(\lambda) - n\lambda - \sum_{i=1}^{n} ln(x_i!)\right] = \\ \left[\frac{1}{\lambda}\sum_{i=1}^{n} x_i - n\right] = 0 \end{aligned} \tag{16}$$

$$\lambda = \frac{1}{\lambda}\sum_{i=1}^{n} x_i \tag{17}$$

Equation 17 confirmed that poisson probability distribution's parameter (p) is the mean of the natural logarithm of rainfall intensity. Table 7 present values of poisson probability distribution's parameter obtained using MLM and MES, and the performance of these methods compared with the standard Weibull method. Tables 8 and 9 provide information on statistical analysis (ANOVA) of the parameters and statistical evaluations of the two methods. These Tables confirmed that the values of the parameter were between 0.754 and 1.695 for both MES and MLM estimator methods. These values of the parameter were similar to the values obtained in literature such as Pisarenko et al. (2002), Vivekanandan (2013), Pichon (2018) and Chacko and Mohan (2018). Outputs from the ANOVA for these parameters (Table 8) confirmed that there was a significant difference between these parameters obtained using the two estimators and methods at a 95% confidence level ($F_{1,2}$ = 564.098 and p = 0.00177, which is less than 0.05).

Figs. 6 and 7 established that the exponential distribution is a continuous distribution as the probability did not discontinue between certain rainfall intensities for both Abeokuta and Makurdi data. Unlike the Bernoulli distribution in which the probability of the rainfall intensity discontinued for the estimator using the MES method between 104.54 mm/h and 124.82 mm/h. These lower performances of this parameter by MES are similar to the performance of negative binomial distribution (Barnett et al., 2006; Rinne, 2008). In addition, the lower performance of the MES method can be attributed to the weak relationship between Weibull probability and Exponential distribution (Eggermont et al., 2009; Ward and Ahlquist, 2018; Wentzel-Larsen and Anhøj, 2019).

Table 7 also confirmed that the relative error was between 0.659 and 1.141, MSC was between −0.028 and 1.339, the values of CD were between 0.118 and 0.682 and R was between 0.344 and 0.826. From these values of relative errors, MSC, CD and R , MLM predicted the Weibull probability better than the MES, based on lower error and higher MSC, CD and R. Tables 8 to 11 present the outputs from ANOVA conducted on the statistical evaluation of effects of selected factors on exponential distribution. Table 9 confirmed that locations had no significant effects on these parameters obtained using the two methods at a 95% confidence level ($F_{1,2}$ = 0.0070 and

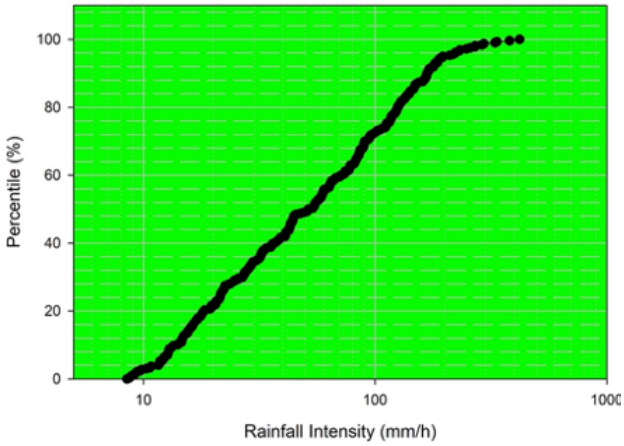

Figure 7. Percentile of rainfall intensities (Abeokuta)

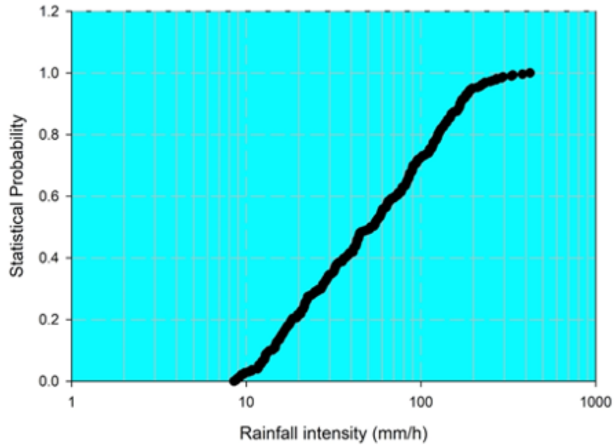

Figure 8. Statistical probabilities of rainfall intensities (Abeokuta)

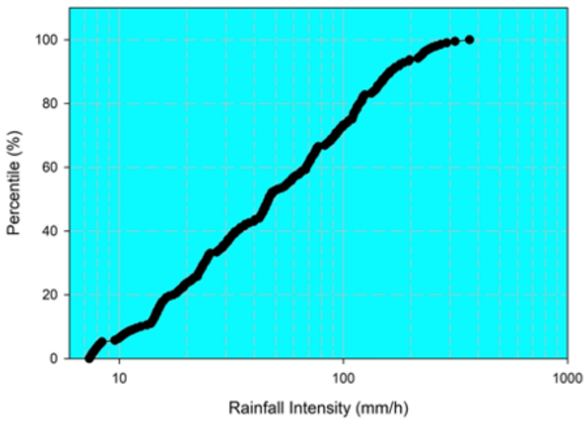

Figure 9. Percentile of rainfall intensities (Makurdi)

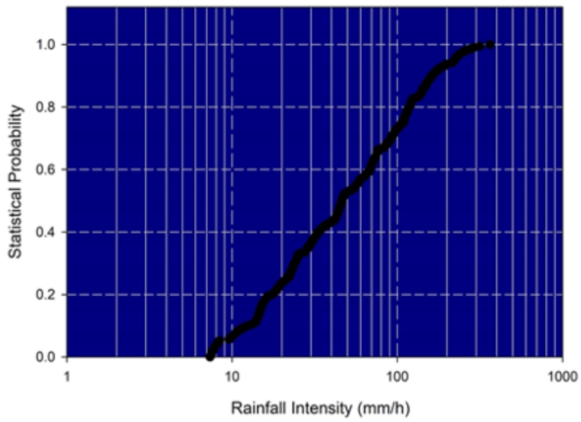

Figure 10. Statistical probabilities of rainfall intensities (Makurdi)

Table 7. Values of Poisson probability distribution's parameter and performance of these methods compared with standard Weibull method

Summary		Makurdi	Abeokuta
Parameter	MLM	1.665	1.695
	MES	1.112	1.148
Error	MLM	0.659	0.743
	MES	1.008	1.141
MSC	MLM	1.044	1.339
	MES	-0.028	0.303
CD	MLM	0.682	0.64
	MES	0.221	0.118
R	MLM	0.826	0.8
	MES	0.47	0.344

Table 8. Effects of the methods on the Poisson distribution parameters

SoV	SS	Df	MSS	F-Value	P-value
Between methods	2.111383	7	0.30162608	19.30639	0.000204
Within methods	0.124985	8	0.01562312		
Total	2.236368	15			

SoV: Sources of variation; SS: Df: degrees of freedom; Sum of square; MSS: Mean sum of square

Table 9. Effects of the Locations on the Poisson distribution parameters

SoV	SS	Df	MSS	F-Value	P-value
Between locations	0.018595	1	0.01859451	0.11738	0.736983
Within groups	2.217773	14	0.15841236		
Total	2.236368	15			

SoV: Sources of variation; SS: Df: degrees of freedom; Sum of square; MSS: Mean sum of square

Table 10. Effects of the parameters on the Exponential distribution

SoV	SS	Df	MSS	F-Value	P-value
Between parameters	0.303076	1	0.303076	564.0981	0.001768
Within parameters	0.001075	2	0.000537		
Total	0.304151	3			

SoV: Sources of variation; SS: Df: degrees of freedom; Sum of square; MSS: Mean sum of square

Table 11. Effects of the locations on the Exponential distribution

SoV	SS	Df	MSS	F-Value	P-value
Between locations	0.001064	1	0.001064	0.007024	0.940841
Within locations	0.303086	2	0.151543		
Total	0.304151	3			

SoV: Sources of variation; SS: Df: degrees of freedom; Sum of square; MSS: Mean sum of square

$p = 0.941$, which is greater than 0.05). Table 10 established that the method had significant effects on the exponential distribution of rainfall intensities data at a 95% confidence level ($F7, 8 = 19.306$ and $p = 2.04 \times 10^{-4}$, which is less than 0.05). Table 11 confirmed that locations had no significant effects on the exponential probabilities obtained using the two methods at a 95% confidence level ($F_{1,4} = 0117$ and $p = 0.737$, which is greater than 0.05).

4 Conclusion

It was concluded based on the findings that MLM estimator was better than MES based on the values of MSC, CD, relative error and R. MLM estimator predicted Weibull probability of rainfall intensity better than MES. Utilization of PPD in the estimation rainfall intensity will help in the prediction of rainfall for agriculture in attaining sustainable development goal 2 (zero hunger), goal 15 (life on land) and goal 14 (life below water), help in regulatory measure for flood control and water security management. There is a need to evaluate the MLM estimator and other probability distributions (such as log Normal, Gamma and Exponential distributions), so as to establish their performance in predicting rainfall intensities with a primary objective of achieving sustainable development goal 2 (zero hunger), goal 15 (life on land) and goal 14 (life below water),

Conflict of Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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